A "Current" Statistical Model and Adaptive Algorithm for Estimating Maneuvering Targets

Hongren Zhou* and K.S.P. Kumar†
University of Minnesota, Minneapolis, Minnesota

A "current model" concept for maneuvering targets is proposed in this paper and a modified Rayleigh density is proposed to describe the "current" probability density of target maneuvering acceleration. The physical relation between the state (acceleration) estimate and the mean value of the state noise in the special case discussed here is also pointed out. Based on these two points, an adaptive Kalman filter for the mean and variance of the maneuvering acceleration is given. Some computer simulation results in one- and three-dimensional cases are given. The simulation results show that the proposed adaptive algorithm can estimate well the states of highly maneuvering targets.

Introduction

DURING the past ten or more years, much research on estimating the states of maneuvering targets has been performed. Useful results, both in the statistical model describing maneuvering targets and in the adaptive filtering algorithm, have been obtained. One of the major difficulties in solving this problem has been the estimation of the current acceleration of a highly maneuvering target when the acceleration itself is varying.

In 1970, Singer[1] suggested a zero-mean, time-correlated maneuvering acceleration model. By the Wiener-Kolmogorov whitening procedure, the random maneuvering acceleration of targets became the output of a system model driven by white noise, and from this, the target motion model was established. Since then, this model has been one of the foundations in the problem of state estimation for maneuvering targets. Moose et al.[2,3] realized that it was not reasonable to use a zero-mean statistical model when a target was doing an accelerating maneuver. They proposed a correlated Gaussian noise model with randomly switching mean, and hence took Singer's model a step further.

Since the statistics of maneuvering targets cannot possibly be known exactly, it is necessary to seek an adaptive estimation method, and some adaptive filters have been developed in recent years.[4]

McAulay and Denlinger[5] discussed an adaptive method in which the target maneuvers first were detected by means of statistical decision theory and then two different filters were used to estimate maneuvering and nonmaneuvering states, respectively. Thorp[6] also discussed the problem of detecting target maneuvering and modeled maneuvering target motion by introducing a binary random variable in the target state equation. The optimal estimate was shown to be a weighted combination of two Kalman filter estimates with weights depending on the likelihood ratio for the detection of a maneuver.

Moose et al.[3] and Williams[7] investigated adaptive filtering algorithms based on Bayes' estimate. Such a Bayes-Kalman tree structure seems complicated, since many possible values of target maneuvering acceleration have to be taken into consideration. In the three-dimensional case especially, the different combinations of maneuvering accelerations in three coordinates make the problem difficult.

Kendrick et al.[8] proposed an interactive filter system where an electro-optical system was used to measure the orientation of a target. The target position, velocity, and acceleration were then estimated. In addition to the need for electro-optical equipment, the method also requires radar to provide data on range and its rate, azimuth angle and its rate, and elevation angle and its rate at the same time.

In modeling maneuvering acceleration, some researchers model the unknown target acceleration as a time-correlated stochastic process.[1,3] As to target acceleration probability density, there are several different models.[1,3,8] This paper proposes a "current model" concept in describing the statistical distribution of maneuvering acceleration. The concept is as follows: in each concrete tactical situation, what is of concern is only the "current" probability density of maneuvering acceleration. When a target is maneuvering with a certain acceleration at present, the region of acceleration which can be taken in the next instant is limited, and is around the "current" acceleration. Hence, it is unnecessary to take all possible values of maneuvering acceleration into consideration when modeling the target acceleration probability density. Since the current acceleration is variable, a variable maneuvering acceleration probability density function (a modified Rayleigh density function) whose mean value is the current acceleration is proposed in this paper. The relationship between the mean value and variance of Rayleigh density is used to set up an adaptive algorithm for the variance of the maneuvering acceleration.

When using the Kalman filter to estimate states of targets, some researchers neglected a simple but important fact: that there exists a relationship between the state variable (acceleration) and the state noise, since the maneuvering acceleration of targets is modeled as the output of a system driven by white noise. More precisely, the estimate of state variable (acceleration) is simply the mean value of the state noise (multiplied by a coefficient). This paper takes advantage of this property in tracking maneuvering targets to realize an acceleration mean value adaptive algorithm.

By the use of this algorithm, the estimation of states and the calculation of Kalman gains are put in a unified closed loop. Some computer simulation results in one- and three-dimensional cases are given. The simulation results show that under the condition that only target position is observed, the adaptive algorithm proposed in this paper can estimate the position, velocity, and acceleration of targets well.

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*Graduate Student, Department of Electrical Engineering.
†Professor, Department of Electrical Engineering.

"Current" Probability Density Model of Maneuvering Target

One of the main problems in the tracking of maneuvering targets is the modeling of the unknown target accelerations. This problem consists of two parts. One is the nature of the stochastic process characterizing the target maneuvers: should this be a white or a time-correlated stochastic process? Researchers tend to hold identical views and model the unknown target acceleration as a time-correlated stochastic process. The second is concerned with specifying the probability density of maneuvering acceleration.

As to target acceleration probability density, there are several different models. Singer[1] suggested a zero-mean, approximate uniform distribution model. Some researchers used Gaussian density with zero-mean or nonzero-mean. Kendrick and others[8] proposed an asymmetrical probability density in modeling the normal load acceleration magnitude. Among these models, the concept of randomly switching mean given by Moose et al.[3] and the asymmetrical model suggested by Kendrick et al.[8] are important. Reference 8 permits the inclusion of hard limits on the acceleration magnitude. Combining the merits of these ideas, we propose the concept of "current model" for maneuvering acceleration. On the one hand, in each concrete tactical situation, we only consider the maneuvering possibility of the target at that time and on that spot, i.e., the "current" probability density of maneuvering acceleration. Thus, the region in which maneuvering acceleration may be taken can be reduced. On the other hand, at each instant, a variable maneuvering acceleration probability density function is used to correspond to the variation of current acceleration.

To accommodate the above two considerations, a variable maneuvering acceleration probability density (a modified Rayleigh density function) is proposed.

When the current acceleration of the target is positive, the probability density function is taken as

$$P_r(a) = \frac{(a_{\max} - a)}{\mu^2} \exp\left\{ -\frac{(a_{\max} - a)^2}{2\mu^2} \right\} \quad a < a_{\max} \qquad (1)$$
$$= 0 \quad a \geq a_{\max}$$

where $a_{\max} > 0$ is the known positive acceleration limit of the target, a is the random acceleration of the target and $\mu > 0$ is a constant. The mean value and the variance of the random acceleration a are, respectively

$$E[a] = a_{\max} - \sqrt{\frac{\pi}{2}}\,\mu$$

and

$$\sigma_a^2 = \frac{4-\pi}{2}\mu^2 \qquad (2)$$

When the current acceleration of the target is negative, the probability density function has the form

$$P_r(a) = \frac{(a - a_{-\max})}{\mu^2} \exp\left\{ -\frac{(a - a_{-\max})^2}{2\mu^2} \right\} \quad a > a_{-\max}$$
$$= 0 \quad a \leq a_{-\max} \qquad (3)$$

where $a_{-\max} < 0$ is the known negative acceleration limit of the target and may not have the same absolute value as $a_{\max}$. In this case, the mean value and the variance of the random acceleration a are, respectively

$$E[a] = a_{-\max} + \sqrt{\frac{\pi}{2}}\,\mu$$

and

$$\sigma_a^2 = \frac{4-\pi}{2}\mu^2 \qquad (4)$$

When the current acceleration of the target is zero, the model of maneuvering acceleration is

$$P_r(a) = \delta(a)$$

where $\delta(\cdot)$ is Dirac δ-function.

In order to follow the variation of current acceleration, at each instant, let the mean value of random acceleration a be equal to the current acceleration. Hence by the use of Eqs. (2) and (4), once the current acceleration value is estimated or measured, the current probability density function of maneuvering acceleration at that instant is also completely determined. This kind of variable probability density function is shown in Fig. 1.

Using a nonzero mean-value, time-correlation model, we have

$$\ddot{x}(t) = \bar{a} + a(t)$$
$$\dot{a}(t) = -\alpha a(t) + w(t) \qquad (5)$$

where $a(t)$ is zero-mean colored acceleration noise, $\bar{a}$ is the mean value of maneuvering acceleration considered as a constant in each sampling period, α is the reciprocal of the maneuver (acceleration) time constant, and $w(t)$ is white noise with zero mean and variance $\sigma_w^2 = 2\alpha\sigma_a^2$.

Let $a_1(t) = a(t) + \bar{a}$. Equation (5) can be rewritten as

$$\ddot{x}(t) = a_1(t)$$
$$\dot{a}_1(t) = -\alpha a_1(t) + \alpha\bar{a} + w(t)$$
$$= -\alpha a_1(t) + w_1(t) \qquad (6)$$

where $a_1(t)$ can be regarded as a state variable and $w_1(t)$ as white noise with mean value $\alpha\bar{a}$. This model (6) looks like Singer's model.[1] The difference, however, is that $a_1(t)$ has mean value $\bar{a}$ and $w_1(t)$ has mean value $\alpha\bar{a}$.

It is known from estimation theory that the optimal estimate of the state variable $a_1(t)$ is just the conditional mean value under the entire past history of observation $Y(t)$

$$\hat{a}_1(t) = E[a_1(t)|Y(t)] \qquad (7)$$

As a matter of fact, $\hat{a}_1(t)$ is the current acceleration that can be obtained via a Kalman filter. Using $\hat{a}_1(t)$ to replace the mean value $\bar{a}$ of $a_1(t)$, the relationship between the estimate of the state variable $a_1(t)$ and the mean value of the state noise $w_1(t)$ can be found. In other words, the estimate of

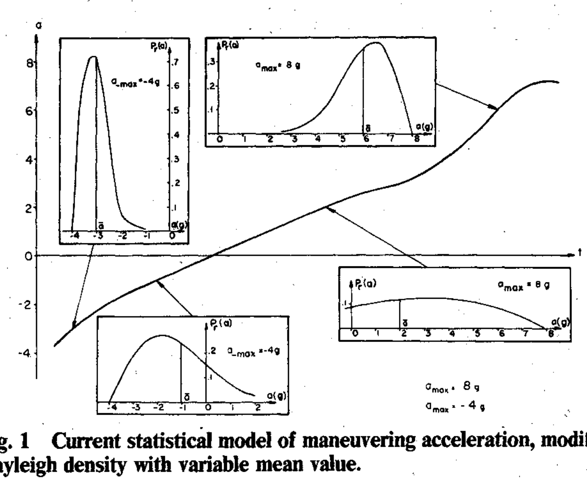

Fig. 1 Current statistical model of maneuvering acceleration, modified Rayleigh density with variable mean value.

the state variable $a_l(t)$ is simply the mean value of the state noise $w_l(t)$, multiplied by a coefficient $1/\alpha$. Furthermore, there exists a relationship between the estimate of the state variable $a_l(t)$ and the variance σ_w^2 of the state noise $w_l(t)$. For example, if the current acceleration is positive, the variance of $w_l(t)$ equals

$$\sigma_w^2 = 2\alpha(4-\pi)/\pi\left(a_{\max} - E[a_l(t)|Y(t)]\right)^2 \tag{8}$$

Adaptive Filtering Algorithm

Considering the nonzero mean value of acceleration, the state equation in the one-dimensional case is

$$\begin{bmatrix} \dot{x}(t) \\ \ddot{x}(t) \\ \dddot{x}(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -\alpha \end{bmatrix} \begin{bmatrix} x(t) \\ \dot{x}(t) \\ \ddot{x}(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \alpha \end{bmatrix} \bar{a} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} w(t) \tag{9}$$

where $x(t)$, $\dot{x}(t)$, and $\ddot{x}(t)$ are the position, velocity, and acceleration of the target, respectively.

When the sampling period is T, after the typical discretizing procedure we obtain the discretized state equation as follows

$$X(k+1) = \Phi(k+1,k)X(k) + U(k)\bar{a} + W(k) \tag{10}$$

where

$$X(k) = [x(k), \dot{x}(k), \ddot{x}(k)]' \tag{11}$$

$$\Phi(k+1,k) = \begin{bmatrix} 1 & T & \frac{1}{\alpha^2}(-1+\alpha T + e^{-\alpha T}) \\ 0 & 1 & \frac{1}{\alpha}(1-e^{-\alpha T}) \\ 0 & 0 & e^{-\alpha T} \end{bmatrix} \tag{12}$$

$$U(k) = \begin{bmatrix} \frac{1}{\alpha}\left(-T + \frac{\alpha T^2}{2} + \frac{1-e^{-\alpha T}}{\alpha}\right) \\ T - \frac{1-e^{-\alpha T}}{\alpha} \\ 1 - e^{-\alpha T} \end{bmatrix} \tag{13}$$

$$W(k) = \int_{kT}^{(k+1)T} \times \begin{bmatrix} \{-1+\alpha((k+1)T-\zeta) + \exp[-\alpha((k+1)T-\zeta)]\}/\alpha^2 \\ \{1-\exp[-\alpha((k+1)T-\zeta)]\}/\alpha \\ \exp[-\alpha((k+1)T-\zeta)] \end{bmatrix} w(\zeta)\,\mathrm{d}\zeta \tag{14}$$

and $W(k)$ is a discrete time white noise sequence[1] with variance $2\alpha\sigma_a^2$.

If only the noisy position data of target are available, the observation equation is

$$Y(k) = H(k)X(k) + V(k) \tag{15}$$

where

$$H(k) = [1,0,0] \tag{16}$$

and $V(k)$ is observation noise, which is Gaussian with zero-mean and variance $R(k)$.

Thus, the standard Kalman filtering equations can be used to produce the state estimates.

According to the statement in the previous section, if we consider the one-step-ahead prediction $\ddot{x}(k+1|k)$ of $\ddot{x}(k)$ as the current acceleration and also as the mean value of randomly maneuvering acceleration at the instant kT, an acceleration mean value adaptive algorithm can be obtained. So let

$$\bar{a}(k) = \hat{\ddot{x}}(k+1|k) \tag{17}$$

and considering Eqs. (12) and (13), we have

$$\hat{X}(k+1|k) = \Phi_l(T)\hat{X}(k|k) \tag{18}$$

where

$$\Phi_l(T) = \begin{bmatrix} 1 & T & T^2/2 \\ 0 & 1 & T \\ 0 & 0 & 1 \end{bmatrix} \tag{19}$$

is just the Newtonian matrix. If there is no acceleration mean value adaptation, matrix $\Phi(k+1,k)$ is approximately equal to $\Phi_l(T)$ only when αT tends to zero. This means that the introduction of mean value adaptive algorithm is almost equivalent to increasing sampling frequency ($T \to 0$) or tracking the target which has very large maneuver acceleration time-constant ($\alpha \to 0$). Both of the cases are advantageous in improving the capability of tracking maneuvering targets.

The computation of state estimation is shown in Fig. 2. It is obvious that

$$\hat{\ddot{x}}(k+1|k) = \hat{\ddot{x}}(k|k)$$

and

$$\hat{\ddot{x}}(k+1|k+1) = \hat{\ddot{x}}(k|k) + K_a(k+1) \times (Y(k+1) - \hat{x}(k+1|k)) \tag{20}$$

where $K_a(k+1)$ is the Kalman gain about acceleration $\ddot{x}(k)$. Equation (20) shows that the correction of the estimate of $\ddot{x}(k)$ from $\hat{\ddot{x}}(k|k)$ to $\hat{\ddot{x}}(k+1|k+1)$ is dominated by two factors: the innovation $\Delta = Y(k+1) - \hat{x}(k+1|k)$ and Kalman gain $K_a(k+1)$. Whether it is necessary to correct $\hat{\ddot{x}}(k|k)$ to get $\hat{\ddot{x}}(k+1|k+1)$ or not depends if innovation Δ is zero or not. And the amount corrected, i.e., the sensitivity of the filter to the innovation Δ, is determined by the Kalman gain $K_a(k+1)$.

As pointed out in the previous section, if we take advantage of the relationship between the estimate $\hat{\ddot{x}}(k|k)$ of the state variable $\ddot{x}(k)$ and the variance σ_w^2 of the state noise [e.g., Eq. (8)], an acceleration variance adaptive algorithm can be accomplished. That is, we let

$$\sigma_a^2 = \frac{4-\pi}{\pi}\left(a_{\max} - \hat{\ddot{x}}(k+1|k)\right)^2 = \frac{4-\pi}{\pi}\left(a_{\max} - \hat{\ddot{x}}(k|k)\right)^2 \tag{21}$$

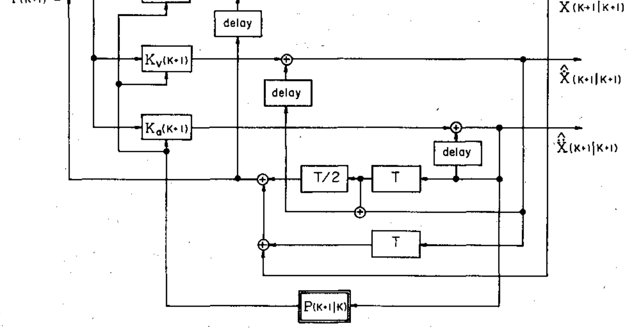

Fig. 2 Block diagram of maneuvering acceleration mean value and variance adaptive algorithm.

when the current acceleration $\hat{\ddot{x}}(k|k)$ is positive, or

$$\sigma_a^2 = \frac{4-\pi}{\pi}\left(-a_{-\max} + \hat{\ddot{x}}(k|k)\right)^2 \tag{22}$$

when the current acceleration $\hat{\ddot{x}}(k|k) < 0$.

From Fig. 2, we can see that by the use of maneuvering acceleration mean value and variance adaptive algorithm, the estimate of the states and the calculation of Kalman gain which are usually computed separately in different channels are put in a unified closed loop.

Computer Simulation

In order to examine the effectiveness of the method proposed in this paper, digital computer simulation was done. In the one-dimensional case, the response of the adaptive system to constant-velocity and constant-acceleration motions of the target were investigated and compared with that of nonadaptive system which is

$$\bar{a} = 0$$

and

$$\sigma_a^2 = \text{const}$$

in the standard Kalman filtering equation.

In the simulation, it was assumed that the variance of the observation noise (or range measuring error) is proportional to the range to target, that is

$$V(k) = (\beta x(k) + \Delta X_0) w(k) \tag{23}$$

where β is a relative error coefficient, ΔX_0 is a fixed measuring error, $w(k)$ is a pseudorandom number which is normal with zero-mean and variance $\sigma^2 = 1$.

So, the variance $R(k)$ of the observation noise is

$$R(k) = (\beta x(k) + \Delta X_0)^2 \cdot E[w^2(k)] \tag{24}$$

The selected parameters in the simulation were: $\Delta X_0 = 30$ m, $\beta = 0.01$, $\alpha = 0.1$, and $T = 1$.

Figures 3 and 4 give the tracks of the velocity and acceleration, respectively, when $X_0 = 30$ km, $V_0 = 300$ m/s, and $a_0 = 0$. The figures show that even in the constant-velocity case the nonadaptive filter has larger variances of estimation errors than the adaptive algorithm.

Figures 5-7 show the estimates of the range, velocity, and acceleration, respectively, when $X_0 = 30$ km, $V_0 = 0$, and $a_0 = 20.0$ m/s^2. It can be seen from the three figures that the nonadaptive algorithm cannot give the correct estimate of acceleration when the target has constant-acceleration motion and the mean errors and the root-mean-square errors of the range, velocity, and acceleration estimates in the nonadaptive case are much larger than those in the adaptive case.

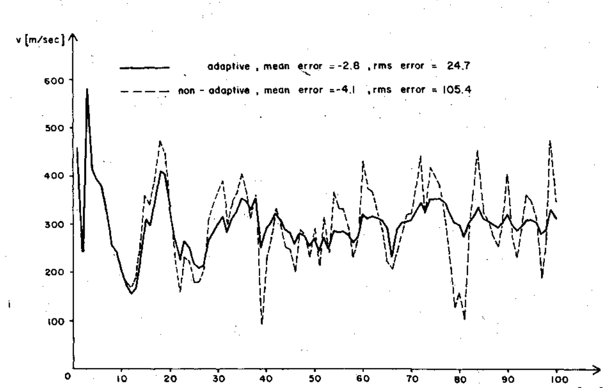

Fig. 3 Velocity estimate of constant-velocity motion target.

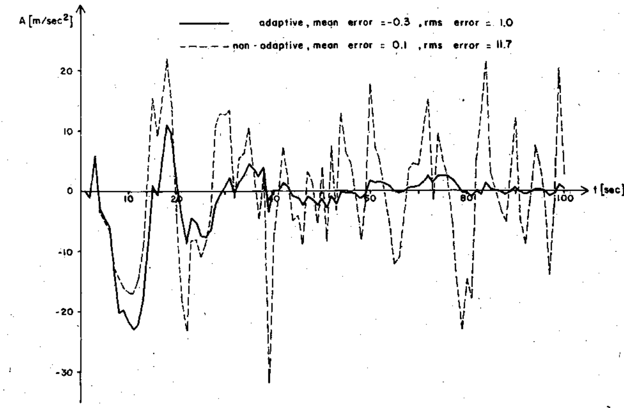

Fig. 4 Acceleration estimate of constant-velocity motion target.

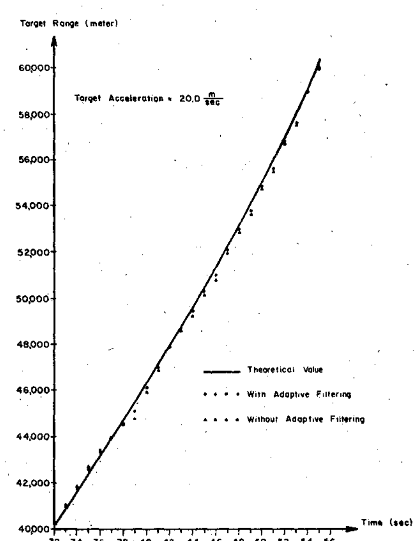

Fig. 5 Range estimate of constant-acceleration motion target.

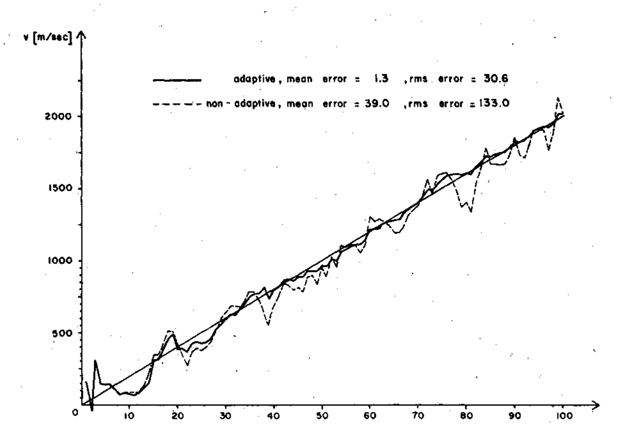

Fig. 6 Velocity estimate of constant-acceleration motion target.

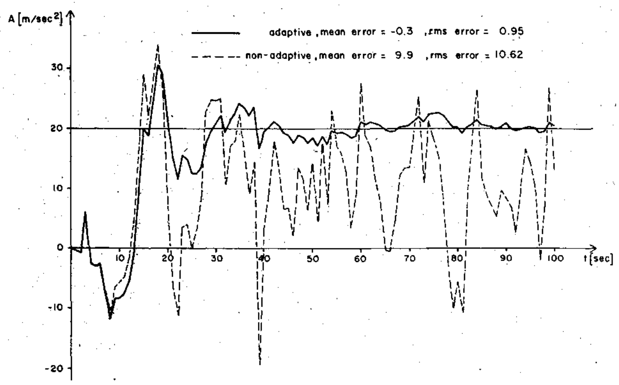

Fig. 7 Acceleration estimate of constant-acceleration motion target.

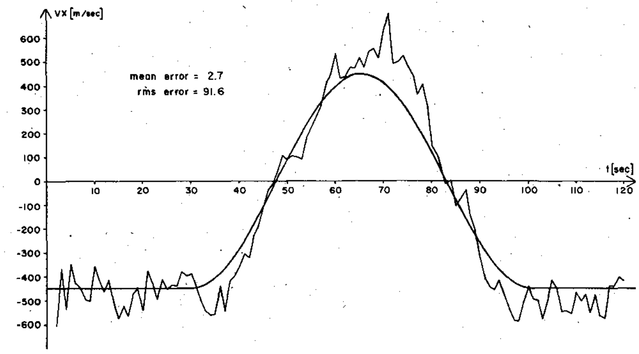

Fig. 10 Velocity estimate in X direction.

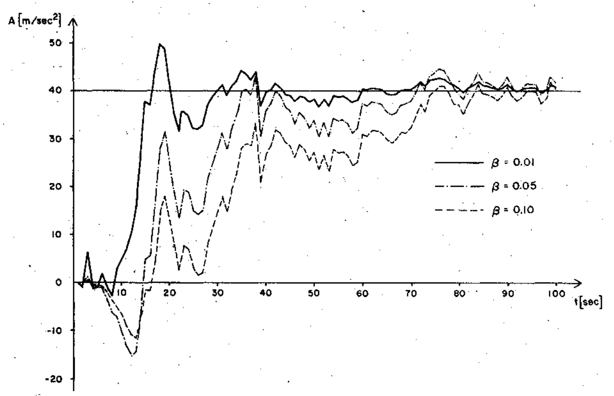

Fig. 8 Influence of relative error coefficient β on acceleration estimate.

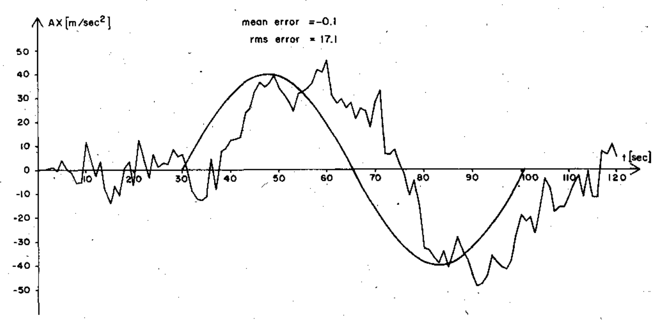

Fig. 11 Acceleration estimate in X direction.

Table 1 Steady-state mean errors and root-mean-square errors of range, velocity, and acceleration estimates (β = 0.01)

Target acceleration, m/s²	Range, m		Velocity, m/s		Acceleration, m/s²	
	me	rmse	me	rmse	me	rmse
0	133.8	219.3	−2.8	24.7	−0.3	1.0
10	175.2	255.4	−0.7	25.1	−0.2	0.89
20	277.7	361.4	1.3	30.6	−0.3	0.95
30	379.4	456.8	3.3	35.1	−0.2	1.0
40	481.9	543.3	5.6	38.9	−0.2	1.0

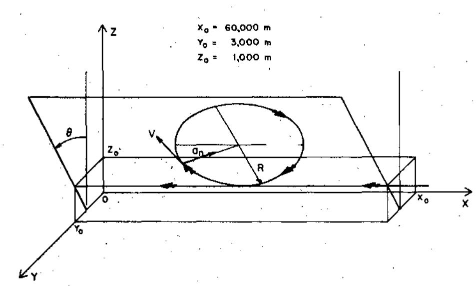

Fig. 9 Normal acceleration maneuver scenario, three-dimensional case.

When $\beta = 0.01$, 0.05, and 0.1, and $a_0 = 40$ m/s² the acceleration estimates are given in Fig. 8 which illustrates the influence of the relative error coefficient β on the estimation error. It shows that even under very bad conditions ($\beta = 0.1$), the adaptive filter still converges.

Table 1 lists the steady-state mean errors (me) and root-mean-square errors (rmse) of the range, velocity, and acceleration estimates corresponding to different target accelerations. Generally speaking, the relative values of the mean errors are less than 0.5% for range, 1% for velocity, and 2% for acceleration, and the relative root-mean-square errors are less than 0.5%, 5%, and 5% for range, velocity, and acceleration, respectively.

Kendrick et al.[8] pointed out that normal maneuvering played a main role in many cases and attempted to use an interactive filter system to estimate the target states in the highly maneuvering case. In order to investigate the quick response capability of the adaptive algorithm to normal maneuver acceleration, a three-dimensional simulation was made. The scenario considered in the simulation is shown in Fig. 9.

In the simulation, it was assumed that the tracking sensor detected the motion of target along the three coordinates independently. Therefore we can extend the one-dimensional model and the algorithm used above to each coordinate. When normal acceleration $A_n = 4.08$ g, $\theta = 30$ deg and target initial velocity $V_0 = -450$ m/s, the results along the X coordinates are shown in Figs. 10 and 11, and the results along the Z coordinate are shown in Figs. 12 and 13. The results along the Y coordinate are similar to those along the Z coordinate. Figure 13 reflects the response capability of the system to an abrupt step acceleration along the Z axis.

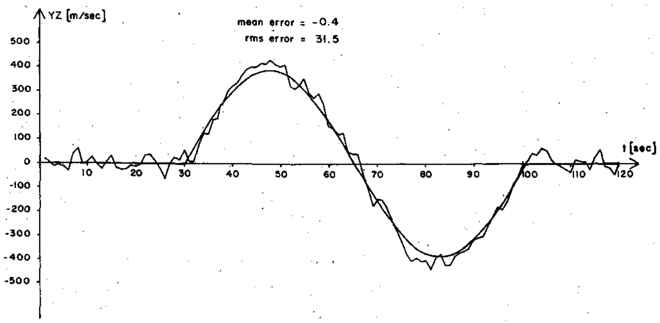

Fig. 12 Velocity estimate in Z direction.

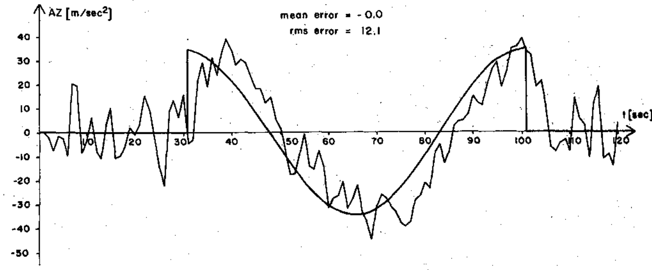

Fig. 13 Acceleration estimate in Z direction.

Table 2 Mean errors and root-mean-square errors of range, velocity, and acceleration estimates in three dimensional case ($\beta = 0.01$)

V, initial velocity, m/s	A_n, normal acceleration, g	R, turning radius, m	T_0 holding time, s	X Range, m me	X Range, m rmse	X Velocity, m/s me	X Velocity, m/s rmse	X Acceleration, m/s² me	X Acceleration, m/s² rmse
	2.04	4,500	94.3	−41.4	319.5	1.8	80.5	−0.7	10.15
−300	4.08	2,250	47.1	−41.6	278.9	1.8	95.8	−0.4	19.2
	6.12	1,500	31.4	−45.2	341.9	−0.7	140.5	−0.2	31.2
	2.04	10,125	141.4	−35.3	273.7	1.0	72.1	−1.1	9.1
−450	4.08	5,062	70.7	−40.9	300.6	2.7	91.6	−0.1	17.1
	6.12	3,375	47.1	−52.8	276.0	−0.8	128.7	−0.6	28.8
	2.04	18,000	188.5	−27.6	224.0	4.3	63.8	−0.0	8.5
−600	4.08	9,000	94.3	−54.8	295.5	0.9	38.4	1.0	14.1
	6.12	6,000	62.8	−44.2	310.2	2.8	124.1	−0.1	26.6

V, initial velocity, m/s	A_n, normal acceleration, g	R, turning radius, m	T_0 holding time, s	Y Range, m me	Y Range, m rmse	Y Velocity, m/s me	Y Velocity, m/s rmse	Y Acceleration, m/s² me	Y Acceleration, m/s² rmse
	2.04	4,500	94.3	−2.7	60.9	0.6	34.8	0.8	9.4
−300	4.08	2,250	47.1	−1.5	50.1	−0.1	32.7	0.5	10.8
	6.12	1,500	31.4	−1.2	49.5	0.6	33.5	0.9	12.6
	2.04	10,125	141.4	−8.8	85.2	−0.5	40.9	0.2	9.7
−450	4.08	5,062	70.7	−1.7	58.9	0.1	33.9	0.6	10.0
	6.12	3,375	47.1	−1.8	52.7	−0.4	34.5	0.4	12.6
	2.04	18,000	188.5	−13.7	103.9	−1.6	44.7	−0.1	9.8
−600	4.08	9,000	94.3	−4.2	75.7	0.9	38.4	1.0	9.9
	6.12	6,000	62.8	−2.2	60.2	−0.4	35.8	0.5	11.9

V, initial velocity, m/s	A_n, normal acceleration, g	R, turning radius, m	T_0 holding time, s	Z Range, m me	Z Range, m rmse	Z Velocity, m/s me	Z Velocity, m/s rmse	Z Acceleration, m/s² me	Z Acceleration, m/s² rmse
	2.04	4,500	94.3	−9.3	51.4	0.7	29.0	0.4	9.1
−300	4.08	2,250	47.1	−3.8	37.1	−0.2	28.1	−0.1	12.7
	6.12	1,500	31.4	−3.8	37.0	0.0	34.7	0.1	17.7
	2.04	10,125	141.4	−16.3	81.9	−1.2	35.9	−0.6	8.9
−450	4.08	5,062	70.7	−9.2	49.9	−0.4	31.5	−0.0	12.1
	6.12	3,375	47.1	−4.5	40.5	−0.8	34.9	−0.3	17.3
	2.04	18,000	188.5	−21.3	104.3	−2.8	39.9	−1.1	8.9
−600	4.08	9,000	94.3	−15.6	73.3	0.6	35.1	0.6	11.3
	6.12	6,000	62.8	−10.0	49.0	−1.4	38.3	−0.3	16.9

Table 2 gives the mean errors and the root-mean-square errors of target position, velocity, and acceleration estimates in X, Y, and Z directions when the target takes different initial velocities and normal accelerations. It can be seen from the table that the model and adaptive algorithm mentioned above can estimate the target states well for the case where the turning radius ranges from 1.5 to 18 km and normal load up to 6.12 g.

Conclusion

The analyses and computer simulation results confirm that the adaptive algorithm proposed in this paper does indeed track highly maneuvering targets. The adaptive algorithm is effective and simple to implement.

The influence of parameter selection on the performance of the filter and extension of the algorithm to estimate the tangential and normal acceleration of maneuvering targets in three dimensions remains to be investigated.

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